

# **HEAT CONDUCTION USING GREEN'S FUNCTIONS**

## **TABLE OF CONTENTS FOR SECOND EDITION** December 2009

	Page
Preface to the first edition	viii
Preface to the second edition	x
Author Biographies	xii
Nomenclature	xiii
<b>1. INTRODUCTION TO GREEN'S FUNCTIONS</b>	<b>1.1</b>
1.1 Introduction	1.1
1.2 Heat Flux and Temperature	1.3
1.3 Differential Energy Equation	1.4
1.4 Boundary and Initial Conditions	1.7
1.5 Integral Energy Equation	1.8
1.6 Dirac Delta Function	1.10
1.7 Steady Heat Conduction in One Dimension	1.12
1.8 GF in the Infinite One-dimensional Body	1.17
1.9 Temperature in an Infinite One-dimensional Body	1.21
1.10 Two Interpretations of Green's Functions	1.26
1.11 Temperature in Semi-Infinite Bodies	1.27
1.12 Flat Plates	1.31
1.13 Properties Common to Transient Green's Functions	1.33
1.14 Heterogeneous Bodies	1.34
1.15 Anisotropic Bodies	1.34
1.16 Transformations	1.35
1.17 Non-Fourier Heat Conduction	1.37
References	1.39
Problems	1.40
Figures, Chapter 1	1.45
<b>2. NUMBERING SYSTEM IN HEAT CONDUCTION</b>	<b>2.1</b>
2.1 Introduction	2.1
2.2 Geometry and Boundary Condition Numbering System	2.2
2.3 Boundary Condition Modifiers	2.5
2.4 Initial Temperature Distribution	2.6
2.5 Interface Descriptors	2.6
2.6 Numbering System for $g(x, t)$	2.7
2.7 Examples of Numbering System	2.8
2.8 Advantages of Numbering System	2.9
References	2.11
Problems	2.12
Figures, Chapter 2	2.13

3.	DERIVATION OF THE GREEN'S FUNCTION SOLUTION EQUATION	3.1
3.1	Introduction	3.1
3.2	Derivation of the One-dimension Green's Function Solution Equation	3.1
3.3	General Form of the GF Solution Equation	3.7
3.4	Alternative Form of the GF Solution Equation	3.15
3.5	Fin term $m^2T$	3.19
3.6	Steady Heat Conduction	3.23
3.7	Moving Solids	3.26
	References	3.32
	Problems	3.32
	Figures, Chapter 3	3.36
4.	METHODS FOR OBTAINING GREEN'S FUNCTIONS	4.1
4.1	Introduction	4.1
4.2	Method of Images	4.2
4.3	Laplace Transform Method	4.3
4.4	Method of Separation of Variables	4.14
4.5	Product Solution for Transient GF	4.22
4.6	Method of Eigenfunction Expansions	4.27
4.7	Steady Green's Functions	4.33
	References	4.42
	Problems	4.43
	Figures, Chapter 4	4.46
5.	IMPROVEMENT OF CONVERGENCE AND INTRINSIC VERIFICATION	5.1
5.1	Introduction	5.1
5.2	Identifying Convergence Problems	5.5
5.3	Strategies to Improve Series Convergence	5.9
5.4	Intrinsic Verification	5.20
	References	5.29
	Problems	5.30
	Figures, Chapter 5	5.32

6.	RECTANGULAR COORDINATES	6.1
6.1	Introduction	6.1
6.2	One-dimension Green's Function Solution Equation	6.2
6.3	Semi-Infinite One-dimension Bodies	6.3
6.4	Flat Plates: Small Cotime Green's Functions	6.11
6.5	Flat Plates: Large Cotime Green's Functions	6.15
6.6	Flat Plates: The Non-Homogeneous Boundary	6.21
6.7	Two-dimension Rectangular Bodies	6.28
6.8	Two-dimension Semi-Infinite Bodies	6.34
6.9	Steady State	6.40
	References	6.44
	Problems	6.45
	Figures, Chapter 6	6.49
7.	CYLINDRICAL COORDINATES	7.1
7.1	Introduction	7.1
7.2	Relations for Radial Heat Flow	7.1
7.3	Infinite Body	7.2
7.4	Separation of Variables for Radial Heat Flow	7.6
7.5	Long Solid Cylinder	7.10
7.6	Hollow Cylinder	7.17
7.7	Infinite Body with a Circular Hole	7.21
7.8	Thin Shells, $T = T(\varphi, t)$	7.23
7.9	Limiting Cases for 2D and 3D Geometries	7.25
7.10	Cylinders with $T = T(r, z, t)$	7.27
7.11	Disk Heat Source on a Semi-Infinite Body	7.30
7.12	Bodies with $T = T(r, \varphi, t)$	7.36
7.13	Steady State	7.41
	References	7.45
	Problems	7.46
	Figures, Chapter 7	7.49
8.	RADIAL HEAT FLOW IN SPHERICAL COORDINATES	8.1
8.1	Introduction	8.1
8.2	Green's Function Solution Equation for Radial Spherical Heat Flow	8.2
8.3	Infinite Body	8.2
8.4	Separation of Variables for Radial Heat Flow in Spheres	8.6
8.5	Temperature in Solid Spheres	8.11
8.6	Temperature in Hollow Spheres	8.24
8.7	Temperature in an Infinite Region Outside a Spherical Cavity	8.27
8.8	Steady State	8.29
	Problems	8.31
	Tables and Figures, Chapter 8	8.34

9.	STEADY-PERIODIC HEAT CONDUCTION	9.1
9.1	Introduction	9.1
9.2	Steady-periodic relations	9.2
9.3	One-Dimensional GF	9.4
9.4	One-Dimensional Temperature	9.9
9.5	Layered Bodies	9.14
9.6	Two- and Three-Dimensional Cartesian Bodies	9.17
9.7	Two-Dimensional Bodies in Cylindrical Coordinates	9.23
9.8	Cylinder with $T = T(r, \varphi, z, \dots)$	9.30
	References	9.35
	Problems	9.37
	Figures, Chapter 9	9.40
10.	GALERKIN-BASED GREEN'S FUNCTIONS AND SOLUTIONS	10.1
10.1	Introduction	10.1
10.2	Green's Functions and Green's Function Solution Method	10.2
10.3	Alternative Form of the Green's Function Solution	10.19
10.4	Basis Functions and simple Matrix Operations	10.23
10.5	Fins and Fin Effect	10.37
10.6	Conclusions	10.39
	References	10.40
	Problems	10.40
	Note 1: Mathematical Identities	10.41
	Note 2: Decomposition of Matrix B	10.42
	Figures, Chapter 10	10.44
11.	APPLICATION OF THE GALERKIN-BASED GREEN'S FUNCTIONS	11.1
11.1	Introduction	11.1
11.2	Basis Function in Some Complex Geometries	11.2
11.3	Heterogeneous Solids	11.10
11.4	Steady-State Conduction	11.17
11.5	Fluid Flow in Ducts	11.20
11.6	Conclusion	11.26
	References	11.26
	Problems	11.27
	Figures, Chapter 11	11.29

12.	UNSTEADY SURFACE ELEMENT METHOD	12.1
12.1	Introduction	12.1
12.2	Duhamel's Theorem and Green's Function Method	12.3
12.3	Unsteady Surface Element Formulations	12.11
12.4	Approximate Analytical Solution (Single Element)	12.25
12.5	Examples	12.30
	References	12.37
	Problems	12.39
	Note 1: Derivation of Equations (12.65a) and (12.65b)	12.40
	Figures, Chapter 12	12.41
APPENDIX B BESSEL FUNCTIONS		B.1
B.1	Generalized Bessel Equation	B.2
B.2	Limiting Form for Small z	B.3
B.3	Limiting Form for Large z	B.5
B.4	Derivatives of Bessel Functions	B.5
B.5	Recurrence Relations	B.5
B.6	Integrals of Bessel Functions	B.6
	References	B.7
APPENDIX D DIRAC DELTA FUNCTION		D.1
D.1	Properties of the Dirac Delta Function	D.1
D.2	Representations of	D.3
D.3	Series Form of	D.3
D.4	Integral Form of $\delta$ and the Fourier Transform	D.5
	References	D.6
APPENDIX E ERROR FUNCTION AND RELATED FUNCTIONS		E.1
E.1	Definition	E.1
E.2	Series Expressions	E.2
E.3	Related Functions	E.2
E.4	Recursion Relation	E.3
E.5	Integrals and Derivatives	E.5
E.6	Complex Argument	E.6
APPENDIX F, FUNCTIONS AND SERIES		F.1
	References	F.2
APPENDIX I, INTEGRALS		I.1
	References	I.13

APPENDIX L	LAPLACE TRANSFORM METHOD	L.1
L.1	Definition	L.1
L.2	Properties of Laplace Transformation	L.4
L.3	Laplace Transform Theorems	L.6
	References	L.7
	Table L.1, Table of Laplace Transforms	L.8
APPENDIX P,	PROPERTIES OF SELECTED MATERIALS	P.1
APPENDIX R,	GREEN'S FUNCTIONS FOR RADIAL CYLINDRICAL COORDINATES	R.1
	References	R.18
	Table R.1,	R.19
APPENDIX R $\Phi$ ,	GREEN'S FUNCTIONS FOR CYLINDRICAL COORDINATES ( $r, \varphi$ )	R $\Phi$ .1
	References	R $\Phi$ .11
APPENDIX $\Phi$ ,	GREEN'S FUNCTIONS FOR CYLINDRICAL POLAR COORDINATE, THIN SHELL CASE	$\Phi$ .1
APPENDIX RS,	GREEN'S FUNCTIONS FOR RADIAL SPHERICAL GEOMETRIES	RS.1
	References	RS.12
APPENDIX X,	GREEN'S FUNCTIONS FOR RECTANGULAR COORDINATES	X.1
	References	X.35
FIGURES FOR ALL APPENDICES		A.1